Module 17

Inverse Laplace Transform and Waveform Synthesis

Objective:
(i) To describe how to obtain inverse Laplace transform making use of the knowledge of properties of Laplace Transform and properties of ROC.
(ii) To apply waveform synthesis to find Laplace forms of certain functions

Introduction:
Inverse Laplace transform maps a function in s-domain back to the time domain. One application is to convert a system response to an input signal from s-domain back to the time domain.

Since system analysis is usually easier in s-domain, the process is to convert the system time domain representation to s-domain (both system and inputs), perform system analysis in s-domain and then convert back to the time domain representation for the response. The reason to do this process in this convoluted way is that due to its properties, the Laplace transform converts the differential equations that describe system behaviour to a polynomial. Also the convolution operation which describes the system action on the input signals is converted to a multiplication operation. These two properties make it much easier to do systems analysis in the s-domain.

Inverse Laplace transform is performed using Partial Fraction Expansion that splits up a complicated fraction into forms that are in the Laplace Transform table.

Description:

Concept of Inverse Laplace Transform

We are aware that the Laplace transform of a continuous signal $x(t)$ is given by

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$$

Since $s=\sigma+j\omega$

$$X(\sigma + j\omega) = \int_{-\infty}^{\infty} x(t)e^{-(\sigma+j\omega)t} dt = \int_{-\infty}^{\infty} \{x(t)e^{-\sigma t}\}e^{-j\omega t} dt = \mathcal{F}\{x(t)e^{-\sigma t}\}$$

$$x(t)e^{-\sigma t} = \mathcal{F}^{-1}\{X(\sigma + j\omega)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\sigma + j\omega)e^{j\omega t} d\omega$$

We can recover $x(t)$ from its Laplace transform evaluated for a set of values of $s=\sigma+j\omega$ in the ROC, with $\sigma$ fixed and $\omega$ varying from $-\infty$ to $+\infty$. Recovering $s(t)$ from $X(s)$ is done by changing the variable of integration in the above equation from $\omega$ to $s$ and using the fact that $\sigma$ is constant, so that $ds=jd\omega$.

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(s)e^{\sigma t}e^{j\omega t} d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(s)e^{\sigma t}d\omega = \frac{1}{2\pi j} \int_{-\infty}^{\infty} X(s)e^{\sigma t} ds$$
The contour of integration in above equation is a straight line in the s-plane corresponding to all points s satisfying \( \text{Re}\{s\}=\sigma \). This line is parallel to the \( j\omega \)-axis. Therefore, we can choose any value of \( \sigma \) such that \( X(\sigma + j\omega) \) converges.

**Partial Fraction Expansion**

As we know that the rational form of \( X(s) \) can be expanded into partial fractions, Inverse Laplace transform can be taken according to location of poles and ROC of \( X(s) \). The roots of denominator polynomial, i.e., poles can be simple and real, complex or multiple.

We know that \( X(s) \) is expanded in partial fractions as

\[
X(s) = \frac{c_0}{s - s_0} + \frac{c_1}{s - s_1} + \frac{c_2}{s - s_2} + \cdots + \frac{c_n}{s - s_n}
\]

Here the roots \( s_0, s_1, s_2, \ldots, s_n \) can be real, complex or multiple. Then the values of \( k_0, k_1, k_2, \ldots, k_n \) constants are calculated accordingly.

In order to find the appropriate time domain function, ROC should be indicated for the \( s \)-domain function. Otherwise we may have multiple time-domain functions based on different possible ROCs.

**Illustration**

**Example for Real roots:**

**Problem 1:** Find out the partial fraction expansion and hence Inverse Laplace transform of the function \( X(s) = \frac{s^2 + 2s - 2}{s(s + 2)(s - 3)} \), ROC: \( \text{Re}\{s\} > 3 \)

**Solution:**

The function \( X(s) = \frac{s^2 + 2s - 2}{s(s + 2)(s - 3)} \) can be written as,

\[
X(s) = \frac{A}{s} + \frac{B}{s + 2} + \frac{C}{s - 3}
\]

The constants calculated are \( A=1/3 \), \( B=-1/5 \), \( C=13/15 \)

\[
X(s) = \frac{1/3}{s} + \frac{1/5}{s + 2} + \frac{13/15}{s - 3}
\]

\[
x(t) = \frac{1}{3} \mathcal{L}^{-1}\left\{ \frac{1}{s} \right\} + \frac{1}{5} \mathcal{L}^{-1}\left\{ \frac{1}{s + 2} \right\} + \frac{13}{15} \mathcal{L}^{-1}\left\{ \frac{1}{s - 3} \right\}
\]

From the given ROC: \( \text{Re}\{s\}>3 \), the resultant signal \( x(t) \) should be right sided.

Therefore, \( x(t) = \frac{1}{3} u(t) + \frac{1}{5} e^{-2t} u(t) + \frac{13}{15} e^{3t} u(t) \)

**Example for Complex roots:**

**Problem 2:** Obtain right sided time domain signal for the function \( X(s) = \frac{s^2 + 2s + 1}{(s + 2)(s^2 + 4)} \)
Solution:

We can write the given function as $X(s) = \frac{s^2 + 2s + 1}{(s+2)(s^2+4)} = \frac{A}{s+2} + \frac{Bs+C}{s^2+2^2}$

The constants can be calculated as $A=1/8$, $B=0.874$, $C=0.5$

Therefore, $X(s) = \frac{1}{8} \frac{1}{s+2} + \frac{0.874s+0.5}{s^2+2^2} = \frac{1}{8} \left( \frac{1}{s+2} \right) + 0.874 \left( \frac{s}{s^2+2^2} \right) + 0.25 \left( \frac{2}{s^2+2^2} \right)$

Finally $x(t) = \frac{1}{8} e^{-2t} u(t) + 0.874 \cos(2t) u(t) + 0.25 \sin(2t) u(t)$

Example for Multiple roots:

Problem 3: Find out the inverse Laplace transform of $X(s) = \frac{s-2}{s(s+1)^3}$, ROC: Re{s} < -1

Solution:

We can write the given function as $X(s) = \frac{s-2}{s(s+1)^3} = \frac{A}{(s+1)^3} + \frac{B}{s(s+1)^2} + \frac{C}{s+1} + \frac{D}{s}$

The constants can be calculated as $A=3$, $B=2$, $C=2$, $D=-2$

Therefore, $X(s) = 3 \left( \frac{1}{(s+1)^3} \right) + 2 \left( \frac{1}{(s+1)^2} \right) + 2 \left( \frac{1}{s+1} \right) - 2 \left( \frac{1}{s} \right)$

From the given ROC: Re{s} < -1, the resultant signal $x(t)$ should be left sided.

Finally $x(t) = 3 \mathcal{L}^{-1} \left( \frac{1}{(s+1)^3} \right) + 2 \mathcal{L}^{-1} \left( \frac{1}{(s+1)^2} \right) + 2 \mathcal{L}^{-1} \left( \frac{1}{s+1} \right) - 2 \mathcal{L}^{-1} \left( \frac{1}{s} \right)$

From the result of Laplace transform

$$\frac{-t^n}{n!} e^{-at} u(-t) \leftrightarrow \frac{1}{(s+a)^{n+1}}, \quad ROC: Re{s} < -a$$

i.e. $x(t) = 3 \frac{-t^2}{2} e^{-t} u(-t) - 2t e^{-t} u(-t) - 2e^{-t} u(-t) + 2u(-t)$

Laplace transform using Waveform Synthesis

In waveform synthesis, the unit step function $u(t)$ and other functions serve as building blocks in constructing other waveforms. Once the waveforms are synthesized in the form of other functions, Laplace transform is found and simplified.

Illustration

For example, we may describe a pulse waveform in terms of unit step functions. A pulse of unit amplitude from $t=a$ to $t=b$ can be formed by taking the difference between the two step functions
i.e., $x(t) = u(t-a) - u(t-b)$

Hence $X(s) = e^{-sa} \frac{1}{s} - e^{-sb} \frac{1}{s}$

Examples:

**Solved Problems:**

**Problem 1:** Find the Inverse Laplace transform of $X(s) = \frac{3s+7}{s^2-2s-3}$ ROC: $\text{Re}\{s\} > 3$

**Solution:**

We know that $e^{-at} u(t) \leftrightarrow \frac{1}{s+a}$, $\text{ROC: Re}\{s\} > -a$

Writing $X(s)$ in the form of partial fraction expansion

$$X(s) = \frac{A}{s-3} + \frac{B}{s+1}$$

The constants can be calculated as $A=4$, $B=-1$

Therefore, $X(s) = \frac{A}{s-3} + \frac{B}{s+1} = 4 \left\{ \frac{1}{s-3} \right\} - \left\{ \frac{1}{s+1} \right\}$

From the given ROC: $\text{Re}\{s\} > 3$, the resultant signal $x(t)$ should be right sided.

i.e., $x(t) = 4 \mathcal{L}^{-1} \left\{ \frac{1}{s-3} \right\} - \mathcal{L}^{-1} \left\{ \frac{1}{s+1} \right\} = 4e^{3t}u(t) - e^{-t}u(t)$

**Problem 2:** Find the Inverse Laplace transform of $X(s) = \frac{-3}{(s+2)(s-1)}$ ROC: $\text{Re}\{s\} < -2$

**Solution:**

We know that $-e^{-at} u(-t) \leftrightarrow \frac{1}{s+a}$, $\text{ROC: Re}\{s\} < -a$

Writing $X(s)$ in the form of partial fraction expansion

$$X(s) = \frac{A}{s+2} + \frac{B}{s-1}$$

The constants can be calculated as $A=1$, $B=-1$

Therefore, $X(s) = \frac{A}{s+2} + \frac{B}{s-1} = \left\{ \frac{1}{s+2} \right\} - \left\{ \frac{1}{s-1} \right\}$
From the given ROC: Re\{s\}<-2, the resultant signal x(t) should be left sided.

i.e., \( x(t) = \mathcal{L}^{-1}\left\{ \frac{1}{s+2} \right\} - \mathcal{L}^{-1}\left\{ \frac{1}{s-1} \right\} = -e^{-2t}u(-t) + e^{t}u(-t) \)

**Problem 3:** Find the Inverse Laplace transform of \( X(s) = \frac{1}{s^2+3s+2} \) ROC: -2<Re\{s\}<-1

**Solution:**

We know that

\[
e^{-at}u(t) \leftrightarrow \frac{1}{s+a}, \quad ROC: Re\{s\}>-a
\]

\[
-e^{-at}u(-t) \leftrightarrow \frac{1}{s+a}, \quad ROC: Re\{s\}<-a
\]

Writing \( X(s) \) in the form of partial fraction expansion

\[
X(s) = \frac{A}{s+1} + \frac{B}{s+2}
\]

The constants can be calculated as \( A=1, B=-1 \)

Therefore, \( X(s) = \frac{A}{s+1} + \frac{B}{s+2} = \left\{ \frac{1}{s+1} \right\} - \left\{ \frac{1}{s+2} \right\} \)

From the given ROC: -2<Re\{s\}<-1, the two derived conditions are \( Re\{s\}<-1 \) which suits for \( \left\{ \frac{1}{s+1} \right\} \) and \( Re\{s\}>-2 \) which suits for \( \left\{ \frac{1}{s+2} \right\} \). Therefore, the resultant signal x(t) should be two-sided.

i.e., \( x(t) = \mathcal{L}^{-1}\left\{ \frac{1}{s+1} \right\} - \mathcal{L}^{-1}\left\{ \frac{1}{s+2} \right\} = -e^{-t}u(-t) - e^{-2t}u(t) \)

**Problem 4:** Determine right-sided \( x(t) \) if \( X(s) = \frac{1}{s^2(s^2-a^2)} \) using convolution theorem

**Solution:**

The given function is \( X(s) = \frac{1}{s^2(s^2-a^2)} = X_1(s)X_2(s) \)

\( X_1(s) = \frac{1}{s^2} \) and \( X_2(s) = \frac{1}{s^2-a^2} = \frac{1}{2a}\left\{ \frac{1}{s-a} - \frac{1}{s+a} \right\} \) (Done using partial fraction expansion)

\[
x_1(t) = \mathcal{L}^{-1}\left\{ \frac{1}{s^2} \right\} = tu(t)
\]

\[
x_2(t) = \frac{1}{2a}\left[ \mathcal{L}^{-1}\left\{ \frac{1}{s-a} \right\} - \mathcal{L}^{-1}\left\{ \frac{1}{s+a} \right\} \right] = \frac{1}{2a}[e^{at} - e^{-at}]u(t)
\]

From the Convolution property \( x_1(t) \ast x_2(t) \leftrightarrow X_1(s)X_2(s) \)

i.e., \( x(t) = x_1(t) \ast x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)x_2(t-\tau)d\tau = \frac{1}{2a} \int_{0}^{t} \tau\left[ e^{a(t-\tau)} - e^{-a(t-\tau)} \right]d\tau \)

\[
x(t) = \frac{1}{2a}\left[ -\frac{2t}{a} + \frac{1}{a^2}(e^{at} - e^{-at}) \right]; t > 0
\]
Problem 5: Find right sided $x(t)$ if $X(s) = \frac{1+e^{-2s}}{3s^2+2s}$

Solution:

Given $X(s) = \frac{1+e^{-2s}}{3s^2+2s} = \frac{1}{s(3s+2)} + \frac{e^{-2s}}{s(3s+2)} = \frac{1}{2} \left[ \frac{1}{s} - \frac{1}{s+2/3} \right] + \frac{1}{2} e^{-2s} \left[ \frac{1}{s} - \frac{1}{s+2/3} \right]$

Taking inverse Laplace transform and using time shifting property

$$x(t) = \frac{1}{2} \left[ 1 - e^{-\frac{2}{3}t} \right] u(t) + \frac{1}{2} \left[ 1 - e^{-\frac{2}{3}(t-2)} \right] u(t-2)$$

Problem 6: Given $x(t) = e^{-t} u(t)$, find the Inverse Laplace transform of $e^{-3s} X(2s)$

Solution:

From Time scaling property

If $x(t) \xrightarrow{L} X(s)$

then $x(at) \xrightarrow{L} \frac{1}{|a|} X \left( \frac{t}{a} \right)$

with $a=1/2$, $x \left( \frac{t}{2} \right) \xrightarrow{L} \frac{1}{1/2} X \left( \frac{s}{1/2} \right) = 2X(2s)$

Therefore, $\frac{1}{2} x \left( \frac{t}{2} \right) \xrightarrow{L} X(2s)$

Time shifting property states that then $x(t-\tau) \xrightarrow{L} e^{-s\tau} X(s)$. Applying this property to above equation with $\tau=3$,

$$\frac{1}{2} x \left( \frac{t-3}{2} \right) \xrightarrow{L} e^{-3s} X(2s)$$

Applying the transformation to $x(t) = e^{-t} u(t)$ given by above equation,

$$\mathcal{L}^{-1} \{ e^{-3s} X(2s) \} = \frac{1}{2} x \left( \frac{t-3}{2} \right) = \frac{1}{2} e^{\left( \frac{t-3}{2} \right)} u \left( \frac{t-3}{2} \right)$$

Problem 7: Find the steady state response of the following system to unit step excitation

$$H(s) = \frac{s+1}{s^2 + 3s + 2}$$

Solution:

We know that for a linear system output $y(t) = h(t) * x(t)$

As the input is $x(t) = u(t) \xrightarrow{L} \frac{1}{s}$

Output $y(t) = h(t) * u(t) \xrightarrow{L} Y(s) = \frac{H(s)}{s}$

Therefore, $Y(s) = \frac{H(s)}{s} = \frac{s+1}{s(s^2+3s+2)} = \frac{s+1}{s(s+1)(s+2)} = \frac{1}{s(s+2)}$

Using partial fraction expansion
\[ Y(s) = \frac{1}{2} \left( \frac{1}{s} - \frac{1}{s + 2} \right) \]

Therefore, step response \( y(t) = \left\{ \frac{1-e^{-2t}}{2} \right\} u(t) \)

**Problem 8:** Knowing that \( ULT \left[ \frac{dy(t)}{dt} \right] = sY(s) - y(0^-) \)

Solve the differential equation \( \frac{d}{dt} y(t) + 5y(t) = x(t) \), with initial condition \( y(0^+) = -2 \) and input \( x(t) = 3e^{2t}u(t) \)

**Solution:**

The given differential equation is, \( \frac{d}{dt} y(t) + 5y(t) = x(t) \)

Taking Unilateral Laplace transform of above equation

\[ sY(s) - y(0^-) + 5Y(s) = X(s) \]

Substituting initial condition \( y(0^+) = y(0^-) = -2 \) and input \( x(t) = 3e^{2t}u(t) \)

\[ \mathcal{L}\{X(s)\} = 3 \frac{1}{s+2} \]

and applying partial fraction expansion

\[ Y(s) = \frac{3}{(s + 2)(s + 5)} - \frac{2}{s + 5} = \frac{1}{s + 2} - \frac{1}{s + 5} - \frac{2}{s + 5} = \frac{1}{s + 2} - \frac{3}{s + 5} \]

Taking inverse Laplace transform of the above equation

\[ y(t) = e^{-2t}u(t) - 3e^{-5t}u(t) \]

**Problem 9:** Obtain the impulse response of the system shown and hence prove that the system is BIBO stable

![System Diagram]

**Solution:**

Let us transform all the elements to their Laplace equivalents assuming zero initial conditions. The Laplace equivalent circuit is shown below

![Laplace Equivalent Circuit]
Using voltage divider formula we can write,

\[ Y(s) = \frac{1}{sC}X(s) = \frac{1}{sRC + 1}X(s) \]

Therefore, transfer function

\[ H(s) = \frac{Y(s)}{X(s)} = \frac{1}{sRC + 1} = \frac{1}{RC} \left[ \frac{1}{s + \frac{1}{RC}} \right] \]

Taking inverse Laplace transform of above equation, impulse response is

\[ h(t) = \frac{1}{RC} e^{-\frac{t}{RC}} u(t) \]

H(s) has one pole located at \( s = -1/RC \)

This pole lies in left half of the s-plane. Hence the system is both causal and stable.

This can also be proved checking the absolute integrability of impulse response

\[ \int_{-\infty}^{\infty} |h(t)| dt < \infty \]

\[ \int_{-\infty}^{\infty} |h(t)| dt = \frac{1}{RC} \int_{0}^{\infty} e^{-\frac{t}{RC}} dt = \frac{1}{RC} \left[ e^{-\frac{t}{RC}} \right]_{0}^{\infty} = -(e^{-\infty} - e^{0}) = 1 < \infty \]

Hence BIBO stable

**Problem 10:** Obtain the Laplace transform of the triangular pulse as shown below using waveform synthesis

**Solution:**
As shown in the figure above, a ramp of slope ‘A’ starting at t=0 is taken as \( f_1(t) \). The function \( f_2(t) \) is the ramp of slope -2A starting at t=1. When we add \( f_1(t) \) and \( f_2(t) \) we get the signal shown in figure (c). Observe that a negative going ramp of slope \((A-2A=-A)\) starts at t=1. To cancel the part of this ramp after \( t \geq 2 \), a positive going ramp of slope +A in figure (d) is added. Then we get the required triangular pulse of Figure (e).

With the help of step functions, the ramp functions in the figure above can be expressed as follows:

\[
\begin{align*}
f_1(t) &= A \, t \, u(t) \quad \text{(The function \( u(t)=1 \) for \( t>0 \); It indicates that ramp is present only for \( t \geq 0 \))} \\
f_2(t) &= -2A \, (t-1) \, u(t-1) \quad \text{(The function \( u(t-1)=1 \) for \( t>1 \); It indicates that ramp is present only for \( t \geq 1 \))} \\
f_3(t) &= A(t-2) \, u(t-2)
\end{align*}
\]

Therefore, \( f(t) = f_1(t) + f_2(t) + f_3(t) = A \, t \, u(t) - 2A \, (t-1) \, u(t-1) + A(t-2) \, u(t-2) \)

Since \( u(t) \), \( u(t-1) \) and \( u(t-2) \) have values of ‘1’ and they just represent the time shifts and directions of ramp functions, they can be dropped in this expression. Laplace transform of above equation becomes

\[
\mathcal{L}\{f(t)\} = A \left( \frac{1}{s^2} - 2A \frac{e^{-s}}{s^2} + A \frac{e^{-2s}}{s^2} \right) = \frac{A}{s^2} \left( 1 - 2e^{-s} + e^{-2s} \right)
\]

Assignment:

**Problem 1:** Find the inverse Laplace transform of \( X(s) = 4s^2 + 15s + \frac{8}{(s+2)(s+1)} \). Assuming signal is causal.

**Problem 2:** The transfer function of the system is given as \( H(s) = \frac{2}{s+3} + \frac{1}{s-2} \). Determine the impulse response if the system is (i) stable (ii) causal

**Problem 3:** Find the current \( i(t) \) in a series RLC circuit as shown in figure below, when a voltage of 100Volts is switched on across the terminals aa’ at \( t=0 \).
**Problem 4:** The response \( h(t) \) of a linear time invariant system to an impulse \( \delta(t) \), under initially relaxed condition is \( h(t) = e^{-t} + e^{-2t} \). Find the step response of this system.

**Problem 5:** Find the Inverse Laplace transform of \( G(s) = \frac{s}{(s-3)(s^2-4s+5)}, \sigma < 2 \)

**Problem 6:** Given \( f(t) \) and \( g(t) \) as show below:

Express \( g(t) \) in terms of \( f(t) \) and hence find Laplace transform

**Problem 7:** Find the Inverse Laplace transform of \( G(s) = \frac{10s^2}{(s+1)(s+3)}, \sigma > 0 \)

**Problem 8:** Let the Laplace transform of a function \( f(t) \) which exists for \( t > 0 \) be \( F_1(s) \) and the Laplace transform of its delayed version \( f(t - \tau) \) be \( F_2(s) \). Let \( F_1^*(s) \) be the complex conjugate of \( F_1(s) \) with the Laplace variable set \( s = \sigma + j\omega \). If \( G(s) = \frac{F_2(s)F_1^*(s)}{|F_1(s)|^2} \) then Find the inverse Laplace transform of \( G(s) \).

**Problem 9:** Find the Laplace transform of periodic sawtooth waveform using waveform synthesis

**Problem 10:** Consider the LTI system for which we are given the following information:

\[
X(s) = \frac{s + 2}{s - 2}
\]

\( x(t) = 0 ; \ t > 0 \)

and \( y(t) = -\frac{2}{3}e^{2t}u(-t) + \frac{1}{3}e^{-t}u(t) \)

(a) Determine \( H(s) \) and its ROC

(b) Determine \( h(t) \)

**Simulation:**
The command one uses now is \texttt{ilaplace}. One also needs to define the symbols \( t \) and \( s \).

Let's calculate the inverse of the previous function \( F(s) \),

\[
F(s) = \frac{s - 5}{s(s + 2)^2}
\]


\[
\begin{align*}
\text{>> syms t s} \\
\text{>> F=(s-5)/(s*(s+2)^2);} \\
\text{>>ilaplace(F)} \\
\text{ans =} \\
-5/4+(7/2*t+5/4)*exp(-2*t) \\
\text{>> simplify(ans)} \\
\text{ans =} \\
-5/4+7/2*t*exp(-2*t)+5/4*exp(-2*t) \\
\text{>> pretty(ans)} \\
-5/4 + 7/2 \, t \, \text{exp}(-2 \, t) + 5/4 \, \text{exp}(-2 \, t)
\end{align*}
\]

which corresponds to

\[
f(t) = -1.25 + 3.5te^{-2t} + 1.25e^{-2t}
\]

Alternatively, one can write

\[
\text{>>ilaplace((s-5)/(s*(s+2)^2))}
\]

Here is another example.

\[
F(s) = \frac{10(s + 2)}{s(s^2 + 4s + 5)}
\]

\[
\begin{align*}
\text{>> F=10*(s+2)/(s*(s^2+4*s+5));} \\
\text{>>ilaplace(F)} \\
\text{ans =} \\
-4*\text{exp}(-2*t)*\text{cos}(t)+2*\text{exp}(-2*t)*\text{sin}(t)+4
\end{align*}
\]

Another example is shown below

\[
\begin{align*}
\text{>>syms s ;} \\
\text{>>T = 1;} \\
\text{>>F1 = 1 - \text{exp}(-T * s ) + \text{exp}(-3*T * s/2);} \\
\text{>>F2 = s^2 ;} \\
\text{>>F = F1 / F2;} \\
\text{>>f = ilaplace( F )} \\
\end{align*}
\]

\[
f =
\begin{align*}
t - \text{heaviside}(t - 1)*(t - 1) + \text{heaviside}(t - 3/2)*(t - 3/2)
\end{align*}
\]

\[
\text{>>pretty( simplify ( f ) )}
\]

\[
t - \text{heaviside}(t - 1) \, (t - 1) + \text{heaviside}(t - 3/2) \, (t - 3/2)
\]

\[
\text{>>ezplot( f )}
\]

\[
\]
References:


