Chapter 1

Signals

1.1 Introduction

In typical applications of science and engineering, we have to process signals, using systems. While the applications can be varied large communication systems to control systems but the basic analysis and design tools are the same.

In a signals and systems course, we study these tools: convolution, Fourier analysis, z-transform, and Laplace transform. The use of these tools in the analysis of linear time-invariant (LTI) systems with deterministic signals.

For most practical systems, input and output signals are continuous and these signals can be processed using continuous systems. However, due to advances in digital systems technology and numerical algorithms, it is advantageous to process continuous signals using digital systems by converting the input signal into a digital signal. Therefore, the study of both continuous and digital systems is required.

As most practical systems are digital and the concepts are relatively easier to understand, we describe discrete signals and systems rst, immediately followed by the corresponding description of continuous signals and systems.
1.2 Classification of the Signals

Signals can be classified into several categories depending upon the criteria and for its classification. Broadly the signals are classified into the following categories:

1. Continuous, Discrete and Digital Signals
2. Periodic and Aperiodic Signals
3. Even and Odd Signals
4. Complex Symmetry and Complex asymmetry Signals
5. Power and Energy Signals

1.2.1 Continuous-time and Discrete-time Signals

Continuous-Time (CT) Signals: They may be defined as continuous in time and continuous in amplitude as shown in Figure 1.1. Ex: Speech, audio signals etc..

Discrete Time (DT) Signals: Discretized in time and Continuous in amplitude. They may also be defined as sampled version of continuous time signals. Ex: Rail traffic signals.

Digital Signals: Discretized in time and quantized in amplitude. They may also be defined as quantized version of discrete signals.

Figure 1.1: Description of Continuous, Discrete and Digital Signals
1.2.2 Periodic Signals

A CT signal \( x(t) \) is said to be periodic if it satisfies the following condition

\[
x(t) = x(t + T_0)
\]

(1.1)

The smallest positive value of \( T_0 \) that satisfies the periodicity condition Eq.(1.1), is referred as the fundamental period of \( x(t) \). The reciprocal of fundamental period of a signal is fundamental frequency \( f_0 \).

Likewise, a DT signal \( x[n] \) is said to be periodic if it satisfies

\[
x[n] = x[n + N_0]
\]

(1.2)

at all time \( k \) and for some positive constant \( N_0 \). The smallest positive value of \( N_0 \) that satisfies the periodicity condition Eq.(1.2) is referred to as the fundamental period of \( x[n] \).

Note: All periodic signals are ever lasting signals i.e. they start at 1 and end at +1 as shown in Figure 1.2.

![Figure 1.2: A typical periodic signal](image)

Ex.1.1 Consider a periodic signal is a sinusoidal function represented as \( x(t) = A \sin (\omega_0 t + \phi) \) The time period of the signal \( T_0 \) is \( \frac{2\pi}{\omega_0} \).

Ex.1.2 CT tangent wave: \( x(t) = \tan(10t) \) is a periodic signal with period \( T = \frac{\pi}{10} \).

Note: Amplitude and phase difference will not affect the time period. i.e. \( 2 \sin(3t), 4 \sin(3t) \),

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4 \sin (3t + 32) \text{ will have the same time period.}

Ex.1.3 CT complex exponential: \( x(t) = e^{j(2t+7)} \) is a periodic signal with period \( T = \)

Ex.1.4 CT sine wave of limited duration:

\[
x(t) = \begin{cases} 
\sin 4t & 2t \text{ } 2 \\
0 & \text{otherwise}
\end{cases}
\]

is a aperiodic signal.

Ex.1.5 CT linear relationship \( x(t) = 2t + 5 \) is an aperiodic signal.

Note: An arbitrary DT sinusoidal sequence \( x[n] = A \sin (\omega n + \phi) \) is periodic if \( \omega = 2 \pi \) is a rational number.

Ex.1.6 \( x[n] = \cos (4n) \) is a periodic signal whereas \( x[n] = \cos (4n) \)

Ex.1.7 Consider the following signal \( g(t) = 2 \cos (4t) - 4 \sin (5t) \)

Calculating the ratio of the two fundamental periods yields

\[
T_1 = \frac{1}{2} \\
T_2 = \frac{2}{5}
\]

which is a rational number. Hence, the linear combination \( g(t) \) is periodic and its period is \( T = 1=2. \)

Ex.1.8 Consider the following signal \( g_1(t) = 2 \cos (4t) \) \( 4 \cos (3t) \)

is aperiodic signal, because the ratio \( \frac{T_1}{T_2} \) is irrational.

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Exercise Problems

Calculate the time-period of the following signals

1. \( x(t) = \cos(18t) + \sin(10t) \)
2. \( x(t) = \cos \frac{2}{5}t \sin \frac{5}{9}t \)
3. \( x(t) = \sin 6tu(t) \)
4. \( x(t) = e^{j10t} \)
5. \( x_1(t) = \sin \frac{5}{8}t + 2 \)
6. \( x_1(t) = \sin \frac{5}{8}t + 2 \)
7. \( x_2(t) = \sin \frac{7}{8}t + 2 \)
8. \( x_3(t) = \sin \frac{7}{8}t + \sin \frac{5}{8}t \)
9. \( x_4(t) = e^{j(5t+4)} \)
10. \( x_5[n] = e^{j7n=4} + e^{j3n=4} \)
11. \( x_6[n] = \sin \frac{3n}{8} + \cos \frac{63n}{64} \)
12. \( x_7[n] = e^{j7n=4} + \sum_{k=1}^{4} \cos \frac{4n}{7} + \sum_{k=1}^{4} \cos \frac{4n}{7} + \sum_{k=1}^{4} \)
13. \( x_8[n] = \sin \frac{3}{10}n + \cos \frac{7}{11}n \)
14. \( x_9[n] = \sum_{k=1}^{4} (n4k) (n14k) \)
15. \( x_{10}[n] = \cos \frac{8}{3}n \)
16. \( x_{11}[n] = \sin \frac{16}{6}n \)
1.2.3 Even and Odd Signals

Any signal can be called even signal if it satisfies \( x(t) = x(-t) \) or \( x(n) = x(-n) \). Similarly any signal can be called odd signal if it satisfies \( x(t) = -x(-t) \) or \( x(n) = -x(-n) \). Figure 1.2 shows an example of an even and odd signal whereas Figure 1.3 shows neither even nor odd signal. Any
signal \( x(t) \) can be expressed in terms of even component \( x_e(t) \) and odd \( x_o(t) \) component.

\[
x(t) = \frac{1}{2} (x(t) + x(-t)) \\
x(t) = \frac{1}{2} (x(t) - x(-t)) \\
x[n] = \frac{1}{2} (x[n] + x[-n]) \\
x[n] = \frac{1}{2} (x[n] - x[-n])
\]

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Ex.1.9 Even and odd components of unit step function are \( x_e(t) = \frac{1}{2} \) and \( x_o(t) = \frac{1}{2} \text{sgn}(t) \), where \( \text{sgn}(t) \) is called signum function.

Exercise Problems

Calculate the even and odd components of the following signals

1. Find the even and odd parts of the signal \( x(t) = u(t) + 1 \)?
2. Find the conjugate anti-symmetric part of \( x(n) = 1 + j2; 2; j5 \)
3. Suppose even part of a signal \( x(n) \) is \( x_e(n) = (1)\frac{n}{2} \) and total energy in \( x(n) = 5 \), nd energy in odd part?
4. Consider the following signal \( x(t) = 3 \sin \frac{2\pi(n-T)}{5} \) Determine the values of `T' for which the resulting signal is (a) an even function and (b) an odd function of the independent variable `t'.
5. Calculate even and odd parts of the following signals

![Figure 1.4: Neither Even nor Odd Signal](image1)
![Figure 1.5: Neither Even nor Odd Signal](image2)
1.2.4 Energy and Power signals

A signal \( x(t) \) (or) \( x(n) \) is called an energy signal if total energy has a non-zero finite value i.e. \( 0 < E_x < 1 \) and \( P_{\text{avg}} = 0 \).

A signal is called a power signal if it has non-zero finite power i.e. \( 0 < P_x < 1 \) and \( E = 1 \).

A signal can't be both an energy and power signal simultaneously. The term instantaneous power is reserved for the true rate of change of energy in a system. In most cases, when the term power is used it refers to average power i.e, the average rate of energy utilization, a constant quantity independent of time.

All periodic signals are power signals and all finite durations signals are energy signals.

\[
E_x(t) = \lim_{T \to 1} \int_{T}^{+T} |x(t)|^2 \, dt \tag{1.3}
\]

\[
P_x = \lim_{T \to 1} \frac{1}{2T} \int_{T}^{+T} j \, x(t) \, dt \tag{1.4}
\]

\[
E_x(n) = \lim_{N \to \infty} \sum_{n=N}^{+N} |x(n)|^2 \tag{1.5}
\]

\[
P_x = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=N}^{+N} |x(n)|^2 \tag{1.6}
\]

Exercise Problems

1. Consider an energy signal \( x(t) \), over the range \( 3 \leq t \leq 3 \) with energy \( E=12 \)J. Find the energy of the following signals. (a) \( x(3t) \) (b) \( 2x(t) \) (c) \( x(t+4) \) (d) \( x(t) \) (e) \( x(t) = \frac{t}{3} + 2 \)

2. Consider a periodic signal \( x(t) \) with the time period \( T=6 \); and power \( P=4 \)W. Find the power of the following signals. (a) \( x(3t) \) (b) \( 2x(t) \) (c) \( x(t+4) \) (d) \( x(t) \) (e) \( x(t) = \frac{t}{3} + 2 \)

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3. Classify each signal as a power signal, energy signal or neither and compute the signal energy or signal power.
1. \( x(t) = u(t) \)
2. \( x(t) = 1 + u(t) \)
3. \( x(t) = \frac{1}{1+|t|} \)
4. \( x(t) = \frac{1}{1+t^2} \)
5. \( x(t) = 1 + \cos tu(t) \)
6. \( x(t) = \frac{1}{t}; t \leq 1 \)
7. \( x(t) = e^{2t}u(t) \)
8. \( x(t) = e^{-t}u(t) \)
9. \( x(t) = e^{(1-t)}u(1-t) \)
10. \( x(t) = e^{t}u(t-2) \)

### 1.3 Elementary Signals

There are several signals exist in signal processing field. Some signals are mentioned below.

1. **Unit step function** \( u(t) \):

\[
u(t) = \begin{cases} 
1; & t \geq 0 \\
0; & t < 0 
\end{cases}
\]

\( u(t) \) has a discontinuity at \( t=0 \).
\( u(t) \) has value of 1/2 at \( t=0 \). Unit step function may also be expressed as

\[
u(t) = \frac{1}{2} (1 + \text{sgn}(t))
\]  

(1.7)

and

\[
u(t) + u(1-t) = 1
\]

(1.8)

Similarly in the discrete domain

\[
u[n] + u[n] = 1 + [n]
\]

(1.9)

\[
u[n] = u[n-1] + [n]
\]
2. Signum function $\text{sgn}(t)$:

$$\text{sgn}(t) = \begin{cases} 
8 & t > 0 \\
>1 & t < 0 \\
0 & t = 0 
\end{cases}$$

3. Sampling function: This is also called as sinc function of $\text{Sa}(t)$ (Sine over Average). It may be defined as

$$\text{Sa}(t) = \frac{\sin t}{t} \quad \text{or} \quad \frac{\sin t}{t}$$  \quad (1.10)

Figure 1.6: Sampling Function

4. Ramp Function $r(t)$: It may be defined as

$$r(t) = t; \quad \text{or} \quad 0r(t) = tu(t)$$

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5. Rectangular Function \( \text{rect}(t) \):
Rect(t) may be defined as

\[
\text{rect}(t) = 1 \quad ; \quad |t| \leq \frac{1}{2}
\]

6. Unit Impulse function \( \delta(t) \):

Consider a gate function, as \( t \to 0 \) every gate function becomes \( \delta(t) = 0 \).

\[
\delta(t) = \begin{cases} 
1 & ; \quad t = 0 \\
0 & ; \quad \text{otherwise}
\end{cases}
\]

Properties

1. Area under \( \delta(t) = 1 \)

\[
\int_{-\infty}^{\infty} \delta(t) \, dt = 1
\]

(1.11)

2. \( \delta(t) \) is even function i.e.

\[
\delta(-t) = \delta(t)
\]

3. Product Property:

\[
x(t) \cdot \delta(t) = x(0) \cdot \delta(t)
\]

4. Product Property:

\[
x(t) \cdot \delta(t-t_0) = x(t_0) \cdot \delta(t-t_0)
\]

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5. Sifting Property:

\[ x(t): \int_a^b (t - t_0) dt = \begin{cases} 
0 & \text{if } x(t) < a \text{ or } x(t) > b \\
0 & \text{if } t_0 = (a, b) \\
\int_a^b & \text{if } a < t_0 < b \\
\int_a^b & \text{does not exist; } a = t_0 \text{ or } b = t_0 
\end{cases} \]

7. Doublet Function:

\[ \overline{d}(t) = \frac{d}{dt} (t) \]

Doublet function is odd function. \( \overline{d}(t) = - \overline{d}(-t) \).

\[ \int_a^b x(t)^{(n)}(t - t_0) dt = (1)^n d_n x(t)|_{t=0} dt^n \]
Exercise Problems

1. \( \int_{-4}^{0} t^2 + 3t \, dt \)

2. \( \int_{0}^{1} (t - 1) (t - 5) \, dt \)

3. \( \int_{0}^{21} (5t - 6) \, dt \)

4. \( \int_{0}^{3} e^{t} \sin(4(t + 5) (t - 1)) \, dt \)

5. \( \int_{0}^{3} \sin(4t + 2t) (t - 5) \, dt \)

6. \( \int_{0}^{3} (u(t + 6) u(t - 6)) \sin(4t) (t - 5) \, dt \)

7. \( \int_{0}^{21} t (t - 0.5m) \, dt \)

8. \( \cos 2t (t - 0.5) \, dt \)

9. \( \int_{0}^{1} t^2 (t - 2) \, dt \)

10. \( \int_{0}^{10} e^t (t - 2) \, dt \)

11. \( \int_{0}^{10} \cos 2t (t - 0.5) \, dt \)

12. \( \int_{1}^{10} e^{3t} + \cos 2t (t - (t)) \, dt \)

1.4 Basic Operations on Signals

There are mainly three operations performed on a signal viz. Time shifting, Time scaling, and Time reversal.

1.4.1 Time Shifting

\( x(t) \rightarrow x(t - \tau) \)

Time Shifting will not affect area.

\( x(t - \tau) \) ! right shift of \( x(t) \) by \( \tau \).

\( x(t + \tau) \) ! left shift of \( x(t) \) by \( \tau \).
1.4.2 Time Scaling

\[ x(t) \rightarrow x(t) \]

Time scaling will affect area.
If \( t > 1 \), it is called compression of \( x(t) \) by \( t \).
If \( 0 < t < 1 \), it is called expansion of \( x(t) \) by \( t \).

1.4.3 Time Reversal

\[ x(t) \rightarrow x(-t) \]

All these three operations can be applied on a signal i.e. \( x(t + \tau) \).

There are two methods to achieve this and shown in the following example.

Consider the following signal, to draw \( x(3t + 8) \)

Method 1: \( x(t) \rightarrow x(t + 8) \rightarrow x(3t + 8) \rightarrow x(3t + 8) \)
1.5 Express the signals in Step and Ramp Functions

Any signal can be expressed in terms of unit step and ramp signals.

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Exercise Problems

Obtain the following signals for the signals $x_1(t); x_2(t); x_3(t); x_4(t)$ given below.

1. $A(t - 6)$  
2. $A(t + 6)$  
3. $A(t + 2.5)$  
4. $A(3t)$  
5. $A(4t)$  
6. $A(3t + 6)$  
7. $A(8t + 12)$  
8. $A(7t - 17)$

where $A$ represents $x_1(t); x_2(t); x_3(t); x_4(t)$. 

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